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Inverse source problems: support reconstruction, uncertainty principles, and stability

Exercises

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Exercise 1: Suppose $k = 1$. Let f be a single layer source supported on a horizontal line segment of width W centered at $z_1 = (0, 0)$, and let g be the same source, translated vertically by a distance d (i.e., centered at $z_2 = (0, d)$). Specifically, with H denoting the Heavyside or indicator function, and δ the dirac mass:

$$f(x, y) = \frac{1}{\sqrt{W}} H_{|x|<W} \delta_{y=0}\,, \qquad (x, y) \in \mathbb{R}^2\,,$$
$$g(x, y) = \frac{1}{\sqrt{W}} H_{|x|<W} \delta_{y=d}\,, \qquad (x, y) \in \mathbb{R}^2\,.$$

Evaluate the far fields u_f^∞ and $u_g^\infty \in L^2(S^1)$ that are radiated by f and g, respectively. Assume that $d \gg W \gg 1$ and derive an upper bound for

$$\frac{\langle u_f^\infty, u_g^\infty \rangle_{L^2(S^1)}}{\|u_f^\infty\|_{L^2(S^1)} \|u_g^\infty\|_{L^2(S^1)}}\,.$$

Compare your result to the general result from the lecture.

Exercise 2: Study the Matlab Codes that have been provided to you for this Winter School. Modify the Matlab file `DataCompletion.m` to write a Matlab file `FarfieldSplitting.m` that implements the far field splitting method from the lecture (for two well-separated sources). In `DataCompletion.m` I have used a Neumann series to solve the linear system from the lecture. This can also be done for the far field splitting problem. To test your algorithm you can modify the Matlab file `Example1.m` and write a Matlab file `Example3.m` with two well separated point sources.