IPwin2021: Inverse source problems: support reconstruction, uncertainty principles, and stability

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We discuss the inverse source problem for time-harmonic acoustic or electromagnetic waves with compactly supported sources. The goal is to recover useful information on a source from observations of its radiated wave far away from the source, i.e., from observations of the far field radiated by the source.

After recalling some basic facts on the direct source problem, we consider the inverse problem to recover the support of a source from observations of its radiated far field. Since this problem is underdetermined, we describe a generalized solution concept, which is known as the convex source support.

Then we turn to question of stability and regularization, and we analyze the source-to-far field operator that maps arbitrary compactly supported sources to their radiated far fields. We determine the singular value decomposition of this operator, and we use it to introduce spaces of non-evanescent far fields that can be radiated by compactly supported limited power sources and at the same time can be detected by receivers with limited sensitivity. These will tell us something about how to regularize traditional minimum norm solutions to the inverse source problem.

Classical uncertainty principles in signal processing limit the amount of simultaneous concentration of a signal with respect to time and frequency. In the inverse source problem, the far field radiated by a source is its restricted Fourier transform. Using this analogy, we discuss in the last part of these lectures two uncertainty principles for far fields radiated by compactly supported sources. Combining these uncertainty principles with our characterization of non- evanescent far fields we provide a stability analysis for two inverse problems that are closely related to the inverse source problem, namely far field splitting and data completion for far fields. These lectures are based on [1, 2, 3, 4, 5].

References

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