IPwin2021: Small electromagnetic inhomogeneities and scattering vs. non-scattering wave-numbers for the Helmholtz equation

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In my set of three lectures I will cover two topics:

Small electromagnetic inhomogeneities and their asymptotic far-field effects, (two first lectures).

Scattering vs. non-scattering wave-numbers for the Helmholtz equation,(third lecture).

The lectures about small electromagnetic inhomogeneities will cover internal inhomogeneities (see for example [1],[9],[10] and [11]) as well as more recent material about "boundary" inhomogeneities [7]. I shall derive the first non-trivial term in an asymptotic (Rayleigh) expansion and contrast that with the small amplitude asymptotic expansion (Born approximation). Using such asymptotic formulas, I will discuss numerical methods to approximately determine the total volume of the inhomogeneities as well as their individual locations, based on electrostatic boundary measurements [8], [10]. In the second lecture I shall discuss the issue of uniformity of the asymptotic formulas, and some consequences of such uniformity [12],[14]. At this point I shall introduce so-called cloaking by mapping techniques (transformation optics) [5],[13]. In particular I will show how uniform estimates enable one to demonstrate the viability of cloaking-by-mapping schemes (electromagnetic invisibility shields).

The third lectures is somewhat independent of the first two (though it discusses related matters) and in that lecture I will discuss some recent results about non-scattering wave-numbers in the context of the Helmholtz equation and penetrable scatterers [15]. In particular, I will discuss the dependence of the number of such wave-numbers on the geometry of the scatterer. I will also introduce the notion of (interior) transmission eigenvalues and relate the non-scattering wave-numbers to these [2].

Books

[1] Habib Ammari and Hyeonbae Kang. *Polarization and moment tensors: with applications to inverse problems and effective medium theory.* eng. Springer, 2007.

[2] Fioralba Cakoni, David Colton, and Houssem Haddar. *Inverse scattering theory and transmission eigenvalues.* Vol. 88. CBMS-NSF Regional Conference Series in Applied Mathematics. Society for Industrial and Applied Mathematics (SIAM), Philadelphia, PA, 2016.

[3] Victor Isakov. *Inverse problems for partial differential equations.* eng. Springer-Verlag, N.Y, 2006.

[4] Christopher B. Croke et al., eds. *Geometric methods in inverse problems and PDE control.* Vol. 137. The IMA Volumes in Mathematics and its Applications. Springer-Verlag, New York, 2004, pp. x+326.

Survey articles

[5] Allan Greenleaf et al. "Cloaking devices, electromagnetic wormholes, and transformation optics". In: *SIAM Rev.* 51.1 (2009), pp. 3–33.

[6] Gunther Uhlmann. "Electrical impedance tomography and Calderón's problem". In: *Inverse Problems* 25.12 (2009).

Articles

[7] Eric. Bonnetier, Charles. Dapogny, and Michael S. Vogelius. *Small perturbations in the type of boundary conditions for an elliptic operator.* Preprint (2021) to appear on `https://sites.math.rutgers.edu/vogelius/`.

[8] Martin Brühl, Martin Hanke, and Michael S. Vogelius. "A direct impedance tomography algorithm for locating small inhomogeneities". In: *Numer. Math.* 93.4 (2003), pp. 635–654.

[9] Yves Capdeboscq and Michael S. Vogelius. "A general representation formula for boundary voltage perturbations caused by internal conductivity inhomogeneities of low volume fraction". In: *M2AN Math. Model. Numer. Anal.* 37.1 (2003), pp. 159–173.

[10] Yves Capdeboscq and Michael S. Vogelius. "Optimal asymptotic estimates for the volume of internal inhomogeneities in terms of multiple boundary measurements". In: *M2AN Math. Model. Numer. Anal.* 37.2 (2003), pp. 227–240.

[11] D. J. Cedio-Fengya, S. Moskow, and M. S. Vogelius. "Identification of conductivity imperfections of small diameter by boundary measurements. Continuous dependence and computational reconstruction". In: *Inverse Problems* 14.3 (1998), pp. 553–595.

[12] Charles Dapogny and Michael S. Vogelius. “Uniform asymptotic expansion of the voltage potential in the presence of thin inhomogeneities with arbitrary conductivity”. In: *Chin. Ann. Math. Ser. B* 38.1 (2017), pp. 293–344.

[13] R. V. Kohn et al. “Cloaking via change of variables in electric impedance tomography”. In: *Inverse Problems* 24.1 (2008).

[14] Hoai-Minh Nguyen and Michael S. Vogelius. “A representation formula for the voltage perturbations caused by diametrically small conductivity inhomogeneities. Proof of uniform validity”. In: *Ann. Inst. H. Poincaré Anal. Non Linéaire* 26.6 (2009), pp. 2283–2315.

[15] Michael S. Vogelius and J. Xiao. *Finiteness results concerning non-scattering wave numbers for incident plane- and Herglotz waves.* Preprint (2020) `https://sites.math.rutgers.edu/vogelius/papers/46pp.pdf`.