IPwin2021 - A primer on PDE's

Exercises

With reference to the distributed slides

Exercise 1 Consider the **inverse Poisson problem**:
Given $u = Kf \in L^2(\Omega)$ from (1), find $f \in L^2(\Omega)$.
Is this inverse problem well-posed?

Exercise 2 Suppose Ω is open, bounded and smooth and let $0 < a \leq \sigma \leq b < \infty$ (a.e.). Show that for $f \in H^{1/2}(\partial\Omega)$ the Dirichlet problem

$$\nabla \cdot \sigma \nabla u = 0 \text{ in } \Omega,$$
$$u|_{\partial\Omega} = f.$$

has a unique solution $u \in H^1(\Omega)$.
(Hint: consider $\tilde{u} = u - \tilde{f}$ for some $\tilde{f} \in H^1(\Omega)$ with $\tilde{f}|_{\partial\Omega} = f$ and use the previous uniqueness result)

Exercise 3 Show that the Dirichlet-to-Neumann map Λ_σ is well-defined in the weak form. (Hint: The definition must be independent from the specific choice of function v)

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